Model Answer – Math 1-A Mid-Term Exam-I November 2016 Total mark: 15

[1]Complete the following: If f(x) is function, then

(a) f(x) is even if ……………… and its curve is symmetric about ………....

(b) f(x) is continuous at a point m if ………………………………………..

(c) The zero of f(x) is the point of intersection of ………………………….

[2]Find the limits :

(a) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{1-2^{3x}}$ (b) $\lim\limits_{x\to 3} \dfrac{\ln(x-2)}{3-x}$ (c) $\lim\limits_{x\to\infty} \dfrac{x+2^x}{x^2-5^x}$

[3]Find y\` where

(a) $y = 2x^3 - 3\sin 2x$ (b) $y = 3^x.\log x$ (c) $y = 2 + (x+\ln x)^{-8}$

(d) $y = (\cos x)^x + (\cos x)^3$ (e) $x\sec x + y\tan y = 3$

Answer

[1](a) f(x) is even if *f(–x) = f(x)* and its curve is symmetric about *y–axis*.

(b) f(x) is continuous at a point m if $f(m) = \lim\limits_{x\to m^+} f(x) = \lim\limits_{x\to m^-} f(x)$

(c) The zero of f(x) is the point of intersection of *its curve with x–axis.*

--- 3 Marks

[a](a) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{1-2^{3x}} = \dfrac{0}{0} = \lim\limits_{x\to 0} -\dfrac{\sin 2x}{8^x - 1} = \lim\limits_{x\to 0} -\dfrac{\left(\dfrac{\sin 2x}{x}\right)}{\dfrac{8^x-1}{x}} = -\dfrac{2}{\ln 8}$

(b) $\lim\limits_{x\to 3} \dfrac{\ln(x-2)}{3-x} = \dfrac{0}{0} = \lim\limits_{x\to 3} -\dfrac{\ln(1+(x-3))}{x-3} = -1$

(c) $\lim\limits_{x\to\infty} \dfrac{x+2^x}{x^2-5^x} = \dfrac{\infty}{\infty-\infty} = \lim\limits_{x\to\infty} \dfrac{\dfrac{x}{5^x}+\dfrac{2^x}{5^x}}{\dfrac{x^2}{5^x}-1} = \dfrac{0+0}{0-1} = 0$

--- 3 Marks

[3](a) $y\grave{} = 6x^2 - 6\cos 2x$ (b) $y\grave{} = 3^x.\ln 3.\log x + 3^x.\frac{1}{\ln 10}.\frac{1}{x}$

(c) $y\grave{} = 0 - 8(x+\ln x)^{-9}(1+\frac{1}{x})$

(d) $y = (\cos x)^x + (\cos x)^3 = e^{x.\ln\cos x} + (\cos x)^3$

Then $y\grave{} = e^{x.\ln\cos x}[1.\ln\cos x + x\dfrac{-\sin x}{\cos x}] + 3(\cos x)^2(-\sin x)$

(e) $x.\sec x.\tan x + 1.\sec x + y.\sec^2 y.y\grave{} + y\grave{}.\tan y = 0$

Then $y\grave{} = -\frac{x.\sec x.\tan x + \sec x}{y.\sec^2 y + \tan y}$

--- 10 Marks

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Model Answer – Math 1-A Mid-Term Exam-II November 2016 Total mark: 15

[1]Complete the following: If f(x) is function, then

(a) Its derivative f \`(x) is defined by ……………………………………….....

(b) Its graph(curve) is ………………………………………………………...

(c) f(x) is odd if ……………….. and its curve is symmetric about …………

[2]Find the limits :

(a) $\lim\limits_{x\to 0} \dfrac{\log(1+2x)}{1-2^x}$ (b) $\lim\limits_{x\to 0} \dfrac{\sin 3x}{3^x-2^x}$ (c) $\lim\limits_{x\to \infty} \dfrac{x^8+2^x}{x^2+3^x}$

[3]Find y\` where

(a) $y = 4^x - 2\cos 3x$ (b) $y = x^3 . \ln x$ (c) $y = 3 - (x+\sin x^2)^7$

(d) $y = (\tan x)^4 + x^{\sin x}$ (e) $x\cos y + y\sec x = 8$

Answer

[1](a)Its derivative is defined by $f\,`(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

(b) Its graph(curve) is the set $\{(x,y):\ y = f(x)\}$.

(c)f(x) is called odd if *f(–x) = – f(x)* and its curve is symmetric about *the origin.*

--- 3 Marks

[2](a) $\lim\limits_{x\to 0} \dfrac{\log(1+2x)}{1-2^x} = \dfrac{0}{0} = \lim\limits_{x\to 0} -\dfrac{[\log(1+2x)]/x}{(2^x-1)/x} = -\dfrac{2/\ln 10}{\ln 2}$

(b) $\lim\limits_{x\to 0} \dfrac{\sin 3x}{3^x-2^x} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{(\sin 3x)/x}{2^x((\frac{3}{2})^x-1)/x} = \dfrac{3}{\ln\frac{3}{2}}$

(c) $\lim\limits_{x\to \infty} \dfrac{x^8+2^x}{x^2+3^x} = \dfrac{\infty}{\infty} = \lim\limits_{x\to \infty} \dfrac{\frac{x^8}{3^x}+\frac{2^x}{3^x}}{\frac{x^2}{3^x}+1} = \dfrac{0+0}{0+1} = 0$

--- 3 Marks

[3](a) $y\,` = 4^x.\ln 4 + 6\sin 3x$ (b) $y\,` = 3x^2.\ln x + x^3.\frac{1}{x}$

(c) $y\,` = 0 - 7(x+\sin x^2)^6(1+\cos x^2\,.\,2x)$

(d) $y = (\tan x)^4 + x^{\sin x} = (\tan x)^4 + e^{\sin x . \ln x}$

$$Then\ \ y\,` = 4(\tan x)^3(sec^2 x) + e^{\sin x.\ln x}[\cos x\,.\ln x + \sin x\,.\frac{1}{x}]$$

(e) $1.\cos y - x.\sin y\,.\,y\,` + y\,`.\sec x + y.\sec x\,.\tan x = 0$

$$Then\ \ y\,` = \frac{\cos y + y.\sec x\,.\tan x}{x.\sin y - \sec x}$$

--- 10 Marks

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